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Numerical Modelling of Artisanal and Small-scale Mining Production

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ABSTRACT: Empirical models of mineral production rely on the match between real output trends and ideal curves. The modelling approach of this process often faces challenges posed by non-ideal patterns displayed by real yields of the mineral volumes extracted. Investigations on a modelling approach based on the Law of Conservation of Matter (LCM) have shown to provide a realistic-oriented solution to this conflict. To that end, a variant of the Fundamental Equation of Mineral Production (FEMP) is developed and investigated. The variant of the FEMP presented considers the features of the equation when the Production to Reserves Ratio (P/R) has a linear rate of change with time. The compliance of this version of the FEMP with the LCM is established via mathematical induction through a configuration that enables it to model any production trend by discretizing its P/R in piecewise linear segments. A numerical model of artisanal and small-scale mining (ASM) is reviewed. When the model is based on one linear trend for the P/R, the application of the variant of the FEMP to the studied case of ASM displays an asymmetrical profile, different from symmetrical trends like the ones of a parabolic or a Hubbert logistic curve. A main difference between the FEMP-based model and the ones using these curves is that the peak of production is achieved at a different time to that when half of the reserves are produced. The linear rate of growth of the P/R in the model based on the FEMP results in the achievement of the peak of production and the depletion of more than half of the reserves to be processed before the midpoint of the total labor time. Since both the P/R of the logistic and parabolic trends of production studied are nonlinear, to reproduce them the FEMP must be applied via piecewise linear segments matching these nonlinear distributions of P/R. The possibility to use this variant of the FEMP to recreate logistic or parabolic curves of production with time gives the latter the advantage of the upgrade of their original empirical design with the credentials of the LCM that support the FEMP. The nature and scope of the models presented are intended to offer a sensible contribution toward the numerical characterization of production and productivity of the human workforce.

Keywords: Salt; Mineral production; Artisanal mining; Small-scale mining; human work; Law of Conservation of Matter; Mathematical induction; Production to reserves ratio; Parabolic curve; Hubbert curve.

Abbreviations: P/R, Production to Reserves Ratio; EMP, Equations of Mineral Production; FEMP, Fundamental Equation of Mineral Production; ASM, artisanal and small-scale mining; LCM, Law of Conservation of Matter; DGDM, Dirección General de Desarrollo Minero; USPTO, United States Patent and Trademark Office.

I. INTRODUCTION

The artisanal and small-scale mining (ASM), an extractive activity of minerals characterized by the employment of rudimentary processes, has had an unprecedented growth in third-world countries during the last decades [1]. Current estimations point out that in excess of 40 million people are dedicated to this activity on a global scale [2], of which the greatest share comes from India (12 million). In spite of the historical and currently growing importance of this informal industrial activity, especially in terms of the significant amount of human resources dedicated to it [3,4], only scatter quantitative studies has been devoted to the characterization of the mineral production outputs as a direct result of human labor workforce [1, 5]. The estimation of mineral yields due to ASM in these studies relied on proportional contributions of ASM respective to total mineral outputs considered. This situation opens a window of opportunity to explore other options of general quantitative methodologies enabled to be applied to model production of a universal range of minerals extracted via ASM.

When empirical models of mineral production, like the Hubbert curve, are considered to proceed with this modeling, the question of having viable options to counter possible mismatches between such ideal curves and real output trends present in ASM or mining at larger scales immediately comes forward as solutions to handle the expected problematic situation [6].

In that order of ideas, a proposal to make such modelling could be based on the mass-balance principle, or “input- equals output”, stated by Law of Conservation of Matter, which has a long tradition of applications in a variety of fields in engineering and natural or social sciences [7].

Mathematical induction, through several variants of the method (strong, downward, etc.), also has a long tradition of applications for the purpose of achieving demonstrations concerning propositions about natural numbers, which comprises both the realms of pure and applied mathematics [8, 9].

The Equations of Mineral Production (EMP) were formulated via strong mathematical induction to comply with the Law of Conservation of Matter [10, 11]. Generally speaking, they correspond to the application of the mass-balance principle using expressions of reserves, production and Production to Reserves Ratio at given times. When the P/R is a constant at all times (*C(t)= C*), the Fundamental Equation (FEMP) of the set of EMP takes the expression of an exponential function:

$$q(t) = CRo(1-C)^{t-1} \qquad (1)$$

Where

q(t) is the production at time "*t*", with '*t*' given in integers

C is the P/R (in this case, *C= q(1)/Ro*) *Ro* are the initial (total) reserves.

Eq. 1 has resulted useful to model the declining stage of mineral production, when the P/R assumes a near constant value. This situation has led to investigate options to widen the scope of applications of the EMP, since acknowledged models of mineral production, like the Hubbert curve [12], can represent, however ideally, the complete cycle of mineral production. These kinds of numerical model represent the rise, peak and decline stages, and are not limited to model only one stage of the cycle. A former development in this direction has shown that dealing with expressions of the EMP when the P/R is variable with time could offer an alternative to model the complete cycle of mineral production [11]. Following this line of research, a formulation of the expressions of the EMP when the P/R changes linearly with time was codified in Excel with the program RPROD, which was filed early in 2019 before the United States Patent and Trademark Office (USPTO) for patent purposes [13]. This formulation of the FEMP has been applied to the modeling and forecast of metallic mineral production at country [14] and an individual well gas scales [15]. In the next section it will be disclosed a version of the FEMP when the P/R changes linearly with time, followed by the presentation of an example of its application to model a complete cycle of ASM mineral production.

The objectives to be achieved in the development of the present study are:

To find an expression of the FEMP when the P/R changes linearly with time.

To develop a proof of the compliance with the LCM of the new expression of the FEMP.

To apply this variant of the FEMP to a case study of small-scale mineral production.

II. MATERIALS AND METHODS

Case study: artisanal salt production

Let's now study a case of artisanal exploitation of a salt pan in a coastal lagoon area. Despite the advance of modern industrial methods of salt production, this means of mineral exploitation is still practiced, having such a notable resilience throughout human history that can be traced back to the ancient world [16].

It has been established that the drying of one cubic meter of regular sea water in a costal lagoon area delivers an average of around 30 g. of salt [17]. However, it is known that either under certain natural embayment conditions [18, 19], or with appropriate setups of the arms or inlets of waters [20], the concentration of salt from marine coastal sources can be drastically increased to high saturation levels close to those of underground waters.

For the case of groundwater or a highly saturated brine of marine origin, where the concentration usually is in the order of tenfold of the regular one for sea water, the average delivery of salt would be close to 300 g. Then, for the latter case, the drying at crystallizer ponds consisting of square parcels of 6 meters long sides containing 10 cm depth salt pools would be capable to yield close to 1.1 kg of salt. With such a premise, results indicate that 50 kg of salt can be produced daily from a 0.40 acre salt pond (or 1,640 m^2).

So, for a half of a daily journal, 25 kg of salt could be harvested from the drying of a 10 cm depth salt pond limited by a major square with 28.8 m long sides, totaling a plain area of 0.20 acres (or 829.4 m^2). A mid-day break would divide the daily journal in two equivalent portions of four hours long each.

Methodology

The next equation corresponds to an expression of a proxy of Eq. 1, which provides for a proposed variant of the FEMP when the P/R has a linear rate of change. This P/R is featured by *C(t)= at+b*, where "*a*" is the slope and "*b*" is the t-intercept.

$$q(t_i) = C(t_i)R(t_i-1)(1-C(t_{i-1}))^{i-1} = (at_i+b)R(t_i-1)\left(1-(a(t_i-1)+b)\right)^{i-1} \qquad (2)$$

Where

The equation is worked in a piecewise fashion, with segments limited to two consecutive points in time (t_i and t_{i+1}, with $t_{i+1} = 1 + t_i$, and i = 1 or 2).

$R(t_i-1)$ and $R(t_{i+1}-1)$ are constant for the interval, and equal to the reserves available at the time (t_i-1), as given by Eq. 3.

For t_1 =1 and t_2 =2, $Ro = R(t_1-1) = R(1-1) = R(0) = R(t_2-1) = R(2-1) = R(1)$, being *Ro* the initial amount of reserves available to produce.

For $t_1 = 3$ and $t_2 = 4$, $R(t_1-1) = R(t_2-1)$, for what $Ro - q(0) - q(1) - q(2) = R(3-1) = R(2) = R(4-1) = R(3)$ and so forth for the next points in time.

For the application of the LCM to demonstrate the intended properties of this equation, we'll use the principle of balance "input equals output" implied by this law [7], applying it to the system Production-Reserves under study.

Then, the compliance of this equation with the LCM is accomplished as far as for every point in time the depletion of reserves corresponds to the amount of production yielded:

$$R(t_i-1) = Ro - \sum_{k=0}^{t_i-1} q(k) \qquad (3)$$

Where *q(0)=0*.

An alternative presentation of Eq. 3 would be:

$$R(t_i-1) = Ro - \sum_{k=0}^{t_i-2} q(k) - q(t_i-1) = R(t_i-2) - q(t_i-1) \qquad (3.1)$$

Which will be useful to consider for later discussions and analysis.

Next, let's demonstrate that Eq. 2 respects the LCM during two consecutive intervals of time {t_i and t_{i+1}}.

To that end, it is sufficient to show that the LCM works for the first interval of time {1,2}, since the same process is repeated in a similar fashion for the following intervals. Then, assuming $t_i = 1$ and $t_{i+1} = 2$ we get:

$$q(1) = (1a+b)R(0)\left(1-(a(1-1)+b)\right)^{1-1} = (a+b)Ro \qquad (4)$$

$$q(2) = (2a + b)R(0)\left(1 - (a(2-1) + b)\right)^{2-1} = (2a + b)Ro\left(1 - (a + b)\right) \quad (5)$$

The LCM stresses that *q(1)*, the amount of production yielded in $t_1=1$, must subtracted in $t_2=2$ from the initial amount of reserves, which makes the reserves available at $t_2=2$ equal to:

$$R(2-1) = Ro - \sum_{k=0}^{2-1} q(k) = \mathrm{Ro} - q(1) - q(0) = Ro - (\mathrm{a} + \mathrm{b})Ro = Ro\left(1 - (a + b)\right) \quad (6)$$

And since the linear growth of the P/R at *t=2* makes *C(2)= 2a+b*, the production is correctly featured according to the LCM by Eq. 5.

Under such considerations, Eq. 2 provides for a whole model of mineral production that can be built upon the sequential arrangement of yields from two successive points of time.

III. RESULTS AND DISCUSSION

The halves of each daily production of this salt pond, distributed in a four hour time span, was modelled in base of Eq. 2, and resulted in Eq. 7. The linear rate of change of the P/R has a slope of 0.10 and a t-intercept of 0.05. Details of the sequence of salt production following the aforementioned trend can be inspected in Fig. 1.

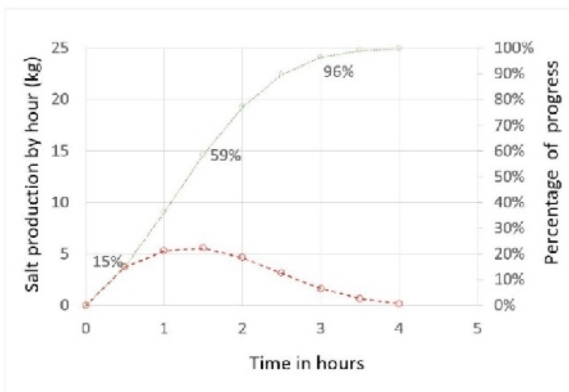

Fig. 1. Salt production model with proxy of the FEMP.

The review of the cumulative production (green points) shows that around 36% of the salt (nine kg) would have been picked by the first hour. By then, the production (red points) amounted slightly in excess of five kg. The peak of salt production (5.6 kg) would occur within the next half hour, when the cumulative yield reaches a total shy of 15 kg. After that, the production rate tends to slow down, and also does the increase of the cumulative production. This is in spite of the linear increase with time of the P/R, because it is operating over ever smaller remaining reserves that already are significantly less than half of the initial volume. As a result, the cumulative production only rises from 19 kg to 24 kg from the second to the third hours. The last hour of production sees a total yield in the order of 1 kg.

In base of Eq. 2, the proxy of the variant of the FEMP applied to make this model is:

$$q(t_i) = (0.1t_i + 0.05)R(t_i - 1)(1 - (0.1(t_i - 1) + 0.05))^{i-1} \quad (7)$$

This equation corresponds to the expression of the general proxy equation (Eq. 2) for the variant of the FEMP when the P/R has a linear rate of change given by *a= 0.1*, and a t-intersect given by *b= 0.05*. For this case, *Ro* would be 25 kg.

In this formulation, a change of the independent variable “working time” (time in half hour fractions from zero to four) is made as *2x* “working time” = *t*. In doing so, every half hour “*t*” is assimilated to an integer between zero and eight.

Discussion: Studies of numerical modeling of mineral production have usually been oriented to the characterization of mining activities with volumes of production in the range of medium to large-scale mining [21, 22]. Regardless of the importance that ASM poses in terms of the number of people employed in many countries where this factor acquires similar dimensions as large-scale mining [3], there is a lack of a comparable volume of research dedicated to model ASM production. In this sense, the contribution of the proposal made about the characterization of small-scale mining provides for a much-needed insight in these kinds of studies that could help tackle many pending issues with regard to ASM, not the least of them being to design strategies toward the formalization of its operations, a priority for many governments [23]. Its relevance could be highlighted by the role that ASM plays in the economies of developing countries, especially as a source of employment and tax revenue from their rural sectors [1, 21]. Another significant aspect tackled by the present study is the provision of a technical frame for the characterization of the production of activities with intrinsic conditions that make difficult their numerical modeling [24].

With the peculiarities given by its piecewise design, the proxy of the FEMP released in Eq. 2 and applied in the case studied (Eq. 7) results in a classic-like trend of rise, peak and decline of production. It is interesting to notice Eq. 2 can be regarded as general if it is bound to the conditions that ensure its compliance with the LCM. This can be seen since Eq. 2 encompasses as a particular case the one when the P/R is constant with time. In this instance, the slope “*a*” of the line would be zero (*a=0*) and for every t_1 present at the start of each piecewise segment of the model, Eq. 2 would adopt the same expression of the exponential function given by Eq. 1:

$$q(t_1) = (0t_1 + b)R\left(1 - (0(t_1 - 1) + b)\right)^{1-1} \rightarrow q(t_1) = bR(1-b)^0 = bRo(1-b)^{t_i - 1} \quad (9)$$

It can be shown that the recalculation of the reserves with time for each piecewise segment [t_i, t_{i+1}] would give the next result:

$$R(t_i - 1) = Ro - \sum_{k=0}^{t_i - 1} q(k) = Ro - bRo - \left(b(Ro - bRo)\right) - .. = Ro(1-b)^{t_i - 1} \quad (10)$$

This result can be proven by strong mathematical induction as follows [25]:

For *n=1*, the hypothesis $R(n-1) = Ro(1-b)^{\mathrm{n}-1}$ is valid since

$R(1-1) = Ro(1-b)^{1-1} = Ro.$

Accepting the hypothesis is right for a natural number m equal to n, meaning $R(m-1) = Ro(1-b)^{\mathrm{m}-1}$, then for *m+1* we'll have:

$$R((m+1)-1) = R(m) = Ro(1-b)^{(\mathrm{m}+1)-1} = Ro(1-b)^{\mathrm{m}}$$

This happens to be true, since by Eq. 9 the production at time *m* equal $q(m) = bR(m-1)$, which by Eq. 3.1 means that:

$$R((m+1)-1) = R(m) = R(m-1) - bR(m-1)$$
$$= Ro(1-b)^{m-1} - bR(m-1)$$
$$= Ro(1-b)^{m-1} - bRo(1-b)^{m-1}$$
$$= Ro(1-b)^{m-1}(1-b) = Ro(1-b)^{m}$$

An aspect to consider is the connection between Eq. 2, and the general expression of the FEMP [11]:

$$q(t) = R(t-1)C(t) \qquad (11)$$

If in Eq. 11, C(t) is represented by the linear function *at+b*, it can be shown that Eq. 2 is equivalent to:

$$q(t) = R(t-1)(at+b) \qquad (12)$$

Which can be deduced from the fact that for every two consecutive natural numbers $\{t_1, t_2\}$, with $t_2 = t_1 + 1$, the combined used of Eq. 3.1 and Eq. 2 results in:

$$q(t_1) = (at_1 + b)R(t_1 - 1)\big(1 - (a(t_i - 1) + b)\big)^{1-1}$$
$$= (at_1 + b)R(t_1 - 1)$$

and

$$q(t_2) = (at_2 + b)R(t_1 - 1)\big(1 - (a(t_2 - 1) + b)\big)^{2-1} =$$
$$(at_2 + b)R(t_1 - 1)\big(1 - (a(t_1 - 1) + b)\big) =$$
$$(at_2 + b)\big(R(t_1 - 1) - R(t_1 - 1)(a(t_1 - 1) + b)\big)$$
$$= (at_2 + b)(R(t_1 - 1) - q(t_1))$$
$$= (at_2 + b)R(t_2 - 1)$$

This not only means that Eq. 2 stands for the variant of the FEMP that considers a P/R that grows linearly with time, but also that can be used to model any production trend by discretizing it in piecewise linear segments. An early investigation about this approach has been applied to model the production of a shale gas well [15].

Respect to the case studied, given the area to be covered and the amount of dedication for each parcel unit within it, the goal of a half working day of salt production seems feasible to be achieved for an artisan after every four hours of labor, as modeled by Eq. 7 or its next equivalent expression made according to Eq. 12:

$$q(t) = R(t-1)(0.1t + 0.05) \qquad (13)$$

The figure of 25 kg of salt by each half of the daily working time (equivalent to around of a ton by month), could even be considered conservative, as it was already mentioned that one laborer can collect over a ton of salt by season from the area that is working [26]. As it was stated, this latter case is likely to happen if every day the worker is harvesting two sets of 25 kg, totaling 50 kg from a salt blanket covering a total area outlined with a square of 41 m long sides. Average annual ASM salt productions up to 58.5 tons by crystallizer ponds portions of 0.40 acres in size (146.2 tons/ac) have been reported from a coastal region in Portugal [27]. In such case, average salinities in excess of 300 ppt can be achieved at the crystallizer ponds where the salt is collected, which is a concentration of a hundredth-fold higher than the one used as premise for the design of the experimental model treated in this article.

Another aspect of interest is that results of studies of productivity by working hours [28, 29], give credibility to the concept of a half daily cycle of production that follows the trend posed Eqs. 7 or 13.

These results show the workers reach a peak of production embedded inside times of lower productivity rates. So, the model made makes more sense as a reality than a simple unrealistic model where workers perform the whole day under a constant rate of production in the order of 25/4 kg/h= 6.3 kg/h.

Although daily outputs that could be significantly lower than 50 kg are to be expected whenever the salt is harvested from ponds like coastal lagoons, which present lower concentrations of salt than those of underground waters, the application of techniques to increase the original low saturation of salts in marine waters have made possible to create salt deposits as rich as those coming from underground waters [27, 30].

An interesting distinctive feature between the model of production provided by the variant of the FEMP in Eq. 2 and the Hubbert curve can be mentioned. The Hubbert curve, which also has been employed to model complete cycles of mineral production, is symmetrical respect a vertical axis passing through its highest point, which is reached when half of the reserves have been produced [12]. The use of this kind of ideal models for the case studied can be inspected in Fig. 2. This figure displays a representation with a logistic curve (red points), which also models the case of salt production previously analyzed with the variant of the FEMP. It can be seen that the peak of production is to be achieved when there has been produced an accumulated volume (green points) of half of the total volume of salt.

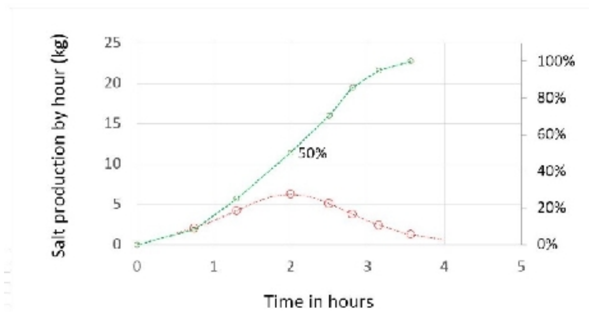

Fig. 2. Salt production model with logistic curve.

For this model, the plateau of production is reached within the half of the total time, with a yield per hour of 6.25 kg/h, which is in the order of a quarter of the total reserves. This is similar to what should be expected in a Hubbert-based model [12, 31, 32], what is not surprising given the symmetry of the curves present in both approaches. The onset of this peak of production occurs simultaneously when half (or 50%) of the cumulative production is achieved (12.5 kg).

For this same case, the curve traced by the FEMP following Eq. 13 shows the noticeable difference of its asymmetry at both sides of the peak of production. In fact, in the example presented in Fig. 1, representing the model with the FEMP, the peak was reached after half of the reserves have been produced. Furthermore, the application of the variant of the FEMP used in this paper displays a trend of raise of production that is different than the one of the decline. In cases where there are reserves large enough to warrant a prolonged continuation of the linear growth of the P/R with time, the late stages of production could even become an asymptotic-like tail that would imply a longer production time span of the resource than one posed by the parabolic curve.

The model of production posed by a parabolic curve (Fig. 3), shows a peak of production starting just before the mid time of 2 hours and then kept after that until 74% of the total volume of salt is accumulated, at the mirror time of the 50% of cumulative production respect to the mid time.

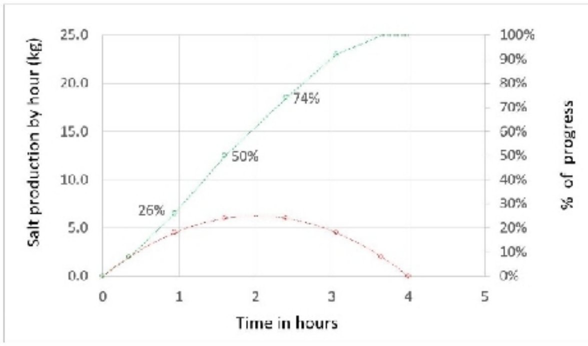

Fig. 3. Salt production model with parabolic curve.

If the investigated variant of FEMP were to be used to reproduce the aforementioned Hubbert and parabolic curves, its application would require to work its P/R in a piecewise fashion, since this is nonlinear, as shown in Figs. 4 and 5.

The P/R corresponding to the logistic or parabolic models can be discretized in several linear segments of positive slope (in blue for the logistic and purple for the parabolic, Figs. 4 and 5, respectively). These distributions can be compared with the segmented lines (in blue for the logistic and purple for the parabolic, Figs. 4 and 5, respectively) that corresponds with the linear fit of the whole set of points. These lines providea reference to figure the nonlinear nature of these distributions.

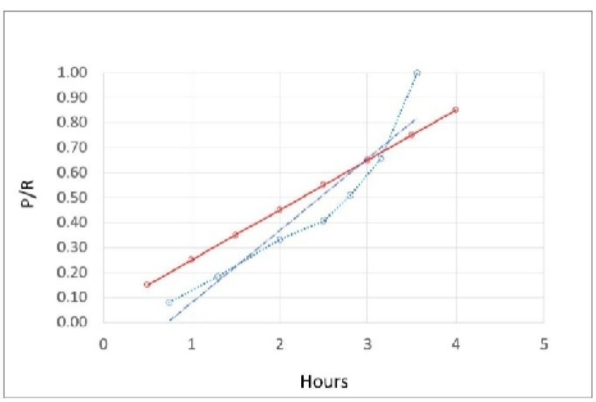

Fig. 4. FEMP and Logistic curves. Distribution in time of their P/R.

An important aspect that arises from the feasibility of the use of a model of mineral production based on the FEMP to reproduce a given logistic or parabolic model is that, since the former is based on the LCM, provides for an upgrade of the scientific credentials of the latter, which were devised by empirical means. Analysis like this opens the possibility to validate empirical models that attempt to reproduce the patterns of mineral production by mimicking its geometrical features, without the support of a fundamental law of nature as in the case of the models based on the FEMP.

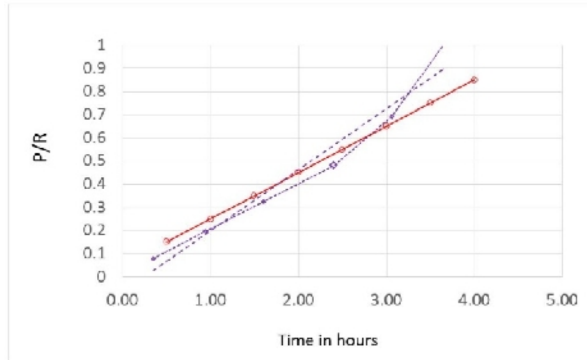

Fig. 5. FEMP and parabolic curves. Distribution in time of their P/R.

When distributions of production outputs do not follow ideal symmetrical patterns, like the ones given by the logistic and parabolic curves, or asymmetrical ones like the model based on linear increase of the P/R with time, the design of Eqs. 2, 11 and 12 provides for the construction of models for such cases, however irregular they could be [14]. In effect, any curve described by a set of points can be discretized in a linear spline fashion by the assembly of consecutive linear segments that constitute secants to the curve at the points in time where yields of production are recorded [33].

The reviewed models succeeded in the replication of the total yield of salt expected to be produced as a result of artisanal labor. However, gathering of further field data is required to address pending questions such as those related to the fair to highest P/R workers can afford, given their working conditions and the rudimentary means available to them.

In these regards, there are evidences that some instances of mineral production could be regulated by an underlying principle where there is a maximum P/R beyond which is not physically viable to increase the rate of depletion of the resource [6].

ASM of salt ponds enjoys the advantage of a relatively easy access and availability of the whole mineral batch to be processed .This advantage is not present during the artisanal production of other minerals, where the pure mineral is scattered disseminated in the rock or unconsolidated material the workers are processing. For the modeling of the production of these minerals, additional considerations are then required to address their particular issues.

Another aspect to consider is the possibility to use Eqs. 2 or 12 to model the whole mineral production of a complex system due to the collective contribution of a group of productive entities (workers, machines, a combination of both), not necessarily operating at the same scale. To that end, it would be suitable a model built upon the addition of the coeval individual contributions to production of each entity, which not necessarily start to produce at the same initial time. Such approach has been used, for instance, with regard to mineral production at large scale modeled via multi-Hubbert cycles [34, 35].

IV. CONCLUSIONS

The contribution of the proposal made about the numerical modeling of ASM provides for a much-needed insight in these kinds of studies, especially noted as it paves the way for significant progress in the solution of critical issues related to this mining activity.

The mineral production at a given time, as provided by the studied variant of the FEMP developed in compliance with the LCM, results of the interaction between the linear change of the P/R with time and the precise amount of remaining reserves present at the start of such time.

This variant of the FEMP can be applied to model the complete cycle of mineral production. For a positive slope and t-intercept the version applied to the case studied generates an asymmetrical concave curve pattern respect to the peak of production. This asymmetry results from reaching such peak at a time different from the one when half of the reserves are produced.

The application of the LCM in this model of mineral production results in a cumulative production at a given time that can also be seen as a percentage of the progress of depletion. This cumulative production authentically records the amount of reserves extracted at a given time from the initial reserves under the conditions of a P/R that linearly changes with time.

The logistic and parabolic curves could also be applied to model a half working day of labor of artisanal salt production. However, these options keep differences with the one based upon the FEMP, notably an idealistic symmetry respect to a plateau of production that is reached precisely at the middle of the working time, when half of the salt or other mineral reserves are about to be collected.

The capability of the investigated variant of the FEMP to fit by linear spline any curve in a piecewise fashion offers a viable alternative to afford a realistic-oriented solution to the conflict posed by mismatches between real and ideal models of mineral yields.

The compliance of the FEMP with the LCM gives to the models of mineral production based on this equation a considerable scientific leverage over those empirical models solely devised upon the geometrical reproduction of patterns of mineral production, and the possibility to validate and even upgrade its scientific credentials with the support of the LCM.

Respect to the case studied, a valid model of salt production from solar evaporation pans based on the FEMP shows how a worker artisan performs a cycle of rise, peak, and decline of production to harvest 25 kg of salt per each one of the two half portions of a working day comprising a total of eight hours of labor. This result implies a monthly collection in the order of a ton, which is feasible and in agreement with documented field reports about artisanal salt mining.

Replications of the total yield of salt expected to be produced as a result of artisanal labor were accomplished through the reviewed models, which are intended to comply with known features of human productivity by working hours. However, gathering of further field data is still required to address pending questions in the subject and to support upgraded models of the process studied as well as models ofASM of other minerals than salt.

V. FUTURE SCOPE

The case studied was focused on modeling mineral production due to an individual worker, the essential component of artisanal work. The coeval addition of the contributions of many components of a productive system beyond artisanal means (workers and machines), modeled via the FEMP, could allow to recreate the global collective output of more sophisticated cases of ASM, even combined with mining beyond that scale. This could be made in a similar fashion that multi-Hubbert cycles are applied to model complex cases of large scale mineral production.

In this sense, the proposed methodology offers a numerical framework that could support the assessment and evaluation of interrelationships between small and larger scale mining operations [36,37].

The numerical assessment of this process of mineral production could also be useful in the studies about the formalization and sustainability of ASM [38].

The compliance of the studied variant of the FEMP with the LCM could be useful for the characterization of other processes beyond the realm of mineral production: agricultural farming [39], industrial manufacturing [40], depletion of monetary reserves [41], and exhaustion of supplies [42]. Also, the suitability of the presented version of the FEMP for linear spline fitting and interpolation has a broad field of potential applications where solutions in the aforementioned fields could be modeled via machine learning and other subfields of artificial intelligence [43, 44].

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